Topic Study Group 38

Task Design and Analysis

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1. Themes and Descriptions the TSG

1.1. Themes of the TSG

In TSG-38, we focused on a variety of theoretical and practical topics related to task design and its analysis. Since there exists a complex, layered relationship between task designers, teachers, and students, which is well-illustrated by successful, theoretically-based long term design research projects, which have resulted in novel materials and approaches impacting teachers and students. Throughout the presentations we actively searched for connections both in regards to methodology, theory and the possible impact of the presented research. We aimed to explore seven related themes: ① frameworks and principles for task design; ② methodological advances for studying task design in mathematics education; ③ relationships between task design, anticipated pedagogies, and student learning; ④ the role of tools in task design; ⑤ task sequences for promoting conceptual understanding and/or higher order thinking skills; ⑥ task design in innovative learning environments; and ⑦ textbook task analysis. We encouraged submissions that offered theoretical and/or empirical contributions and sought to include research from a variety of cultural contexts to enhance our discussions.

1.2. Submissions

We invited two presentations and received a total of 38 submissions from 17 countries (South America 1, North America 2, Asia 7, and Europe: 7), with diverse cultural representation. Of those 38 submissions, thirty-one papers were accepted as paper presentations (fourteen as long oral and seventeen as short oral), five as posters, and two were rejected.

1.3. Paper topics

Of the 31 accepted papers, only 24 papers were able to be presented during the online conference.

A list of these papers and authors are included in order of presentation and are organized with related themes in Tab. 1.

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Tab. 1. List of papers presented

Paper and author(s)	
[1]	Action, process or object? Can they all be perceived in a single task? ***Maria Trigueros****, Asuman Oktaç, Rita Xochitl Vázquez Padilla, and Avenilde Romo Vázquez* (Mexico). ①③⑤
[2]	The design of tasks for automatic formative assessment: Supporting teachers and students. ***Willy Viviani*** *and Kayla White* (USA). ①②③⑥
[3]	A joint embodied and simulation design for graphing: Coordinating distances that change together. ***Heather Lynn Johnson*** (USA)*, Anna Shvarts* (The Nethelands)*, and Amy Smith* (USA). ①②③⑥
[4]	Collective work on task design through study and research path for teacher education. ***Berta Barquero*** *and Sonia Esteve* (Spain). ①②③
[5]	Exploring mathematical task designed by pre-service teachers. ***Ruchi Mittal*** *and Alprata Ahuj* (India). ①⑤
[6]	The fundamental idea of task design in China for algebraic development. ***Xuhua Sun*** (Macao SAR, China). ①③⑤
[7]	Schooling experience as mediating variables in preservice teachers' beliefs and instructional practice when designing mathematical tasks. ***Eugenio Chandia Muñoz*** (Chile). ②③
[8]	Transforming mathematics tasks: an important mathematics teacher's role. ***Guillermina Ávila García****, Liliana Suárez Téllez, and Víctor Hugo Luna Acevedo* (Mexico). ①③⑤
[9]	Developing silent video tasks' instructional sequence. ***Bjarnheidur Kristinsdottir****, Freyja Hreinsdottir* (Iceland)*, and Zsolt Lavicza* (Austria). ②④⑥
[10]	A possible pathway of mathematical inquiry: how to calculate the cube root of a given number by using a simple pocket calculator? ***Koji Otaki****, Hiroaki Hamanaka, and Takeshi Miyakawa* (Japan). ①⑤⑥
[11]	Research on designing and teaching of worked examples in reviewing of sequence based on the SOLO taxonomy. ***Junyi Li*** *and Chao Zhou* (China). ①③⑤
[12]	Fermi problems as a hub for task design in mathematics and stem education. ***Jonas Bergman Ärlebäck*** (Sweden)*and Lluís Albarracín* (Spain). ①②⑤⑥
[13]	Opportunities for inquiry-based learning provided by Chinese and Dutch lower-secondary school mathematics textbook tasks. ***Luhuan Huang****, Michiel Doorman and Wouter van Joolingen* (The Netherlands). ②⑤⑦
[14]	Developing digital mathematical tasks to promote students' higher order thinking skills. ***Meryansumayeka****, Zulkardi, Ratu Ilma Indra Putri, and Cecil Hiltrimartin* (Indonesia). ②⑤⑥
[15]	Potential, actual and practical variations for teaching functions: cases study in China and France. ***Luxizi Zhang*** (China)*, Luc Trouche* (France)*, and Jiansheng Bao* (China). ③⑤⑦
[16]	Students' opportunities to engage in mathematical problem solving. ***Jonas Jäder*** (Sweden). ①⑤⑦
[17]	Tasks and scenarios for promoting inquiry-based mathematics teaching. ***Michiel Doorman*** (The Netherlands)*, Matija Bašić* (Croatia)*, Zeljka Milin Sipus* (Croatia)*, and Rogier Bos* (The Netherlands). ①③⑤
[18]	Towards differentiated instruction: Insights from constructivist learning design. ***Ng Kit Ee Dawn****, Lee Ngan Hoe, Cynthia Seto, Mei Liu, Lee June, and Zi Yang Wong* (Singapore). ①③⑤
[19]	Task for introducing the vector concept using technology. ***Sofia Paz Rodriguez****, Carlos Armando Cuevas Vallejo and Hosé Orozco-Santiago Cinvestav* (Mexico). ①④⑤
[20]	Design tasks in MLR environment: Constructing examples for proving logical statements. ***Galit Nagari-Haddif*** (Israel). ①⑤⑥
[21]	Didactic sequence planning for the study of the teaching and learning of isometries in future primary school teachers. ***Marta Martin Nieto*** *and Natalia Ruiz-Lopez* (Spain). ①③⑤
[22]	Analyzing primary two pupils' errors answering fractions' task using the Newman procedure. ***Rosmawati Mohamed*** *and Munirah Ghazali* (Malaysia). ①⑤
[23]	Effects of low floor high ceiling mathematical tasks on students' mathematical proficiency in seventh-grade geometry. ***Franklin Falculan*** *and Maria Alva Aberin* (Philippine). ①⑤
[24]	Collaborative design of unit that fosters reification of a mathematical object. ***Minoru Ohtani*** (Japan). ①⑤

2. Program Overview

2.1. *Sessions*

There were so many high-quality submissions, the ICMI organizing committee granted our TSG one more time slot for presentations. In our first 90-minute session, the TSG Chair, Minoru Ohtani, introduced the rest of the Team and described the format of the sessions. Generally, all four-time sessions led with 20 minutes *invited long* oral and presentation and discussion and *long* and *short* oral presentations with a 30- and 10-minutes collective discussion respectively. Throughout the four days, we attempted to facilitate participant dialogue to collectively identify emerging research themes, potential interdisciplinary approaches and future research opportunities. We used an online discussion platform, "Padlet," to facilitate communication among participants. We shared roles of online chair, note taker, and onsite organizer, to foster participant networking as well as to counter "virtual conference" fatigue. At the end of our last session, we built in 20 minutes for whole group reflection, discussion, and suggestions for needed research trajectories.

2.2. *Session 1*

In the session 1, we had one invited and three long oral presentations. The invited presentation by Trigueros[1] discussed an example of a task designed from the viewpoint of APOS theory to examine students' understanding of linear transformations. The analysis of students' responses from the lens of APOS theory, coupled with observations made by using the Anthropological Theory of Didactics (ATD) offer important elements that can be used in the redesign of the task. The design of task with APOS theory made it possible for students with different conceptions to start working on the task and obtain an answer that they considered satisfactory. At the same time APOS theory enables researchers to look closely into details involved in students' responses and discern different conceptions. The design of task with ATD evokes the issue of students' interpretation and performance of the task in reference to the social and institutional situations. Viviani and White[2] proposed task design for automatic formative assessment of student responses in a calculus class which provides students with opportunities to engage in exploration to change and develop their mathematical perspectives. Johnson et al.[3] elaborated a joint embodied and simulation design methods which are rooted in different theoretical traditions. The empirical evidence and implications of two phases of joint design method were discussed. Joining genres brings in a correspondent theory and highlights aspects of learning, together providing the rich affordances for covering a variety of possible gaps in mathematics understanding. This approach is particularly relevant for task sequences involving socially shared, and yet spatially articulated, mathematical notations, such as Cartesian graphs. Barquero and Esteve[4] reflected on the work of transposing research and methodological tools to teacher education for primary school teachers' practice of designing and analysing tasks. The case of the study and research paths had

theoretical underpinning of the anthropological theory of the didactic and the research methodology proposed by didactic engineering.

2.3. *Session 2*

In the session 2, we had seven short oral presentations. Mittal and Ahuj[5] investigated the exploration of mathematical task design by pre-service teachers from three aspects: their source, type, and appropriateness, all of which can give holistic understanding to the researchers. Sun[6] delineated the fundamental idea of task design for algebra knowledge development with specific emphasis on Chinese cultural context. Muñoz[7] presented research results on the relationship between the schooling experience by prospective teachers, their instructional beliefs and practices, and the way they design mathematical tasks. The analysis showed a predominant pattern oriented towards constructivist teaching practices, which was influenced by the schooling of the prospective teacher. García et al.[8] presented results of a teacher's task design in combinatorics based on the "mode 5e". Transformation of task sequences with the 5e mode broadened teacher's vision and promoted high school students' perception. Kristinsdottir et al.[9] presented results of design research project on developing silent video tasks in collaboration with researcher, upper secondary teachers, and students. The research showed that the silent video tasks could be used as formative assessment and in classroom discussion. Otaki et al.[10] analyzed an authentic inquiry in didactic situations within the framework of the anthropological theory of the didactic (ATD), especially by the Herbartian schema. Inquiry process by prospective mathematics teachers who engaged with a task about calculation cubic root by pocket calculators was deliberately analyzed by the schema. Li and Zhou[11] reported on SOLO taxonomy-based action research on the design of multiple task examples for high school students. The examples had holistic, hierarchical, and self-explanatory nature and benefit most students at different levels.

2.4. *Session 3*

In the session 3, we had one invited and seven long oral presentations. The invited talk[12] by Ärlebäck discussed research on so called "Fermi problems" and the fundamental principles underlying this type of tasks and their use. Based on the model and modeling perspective on teaching and learning, the research developed the "FPAT-framework" for supporting the design and use of Fermi problems to facilitate not only students' learning mathematics concepts and higher order thinking skills but also interdisciplinary collaborations with other subjects, especially STEM subjects. Luhuan Huang et al.[13] presented a comparative textbook task analysis of lower-secondary schools in Beijing and the Netherlands, using an inquiry-based learning (IBL) framework. The analysis showed that tasks in both textbooks provide some opportunities for IBL in phases related to solution procedures and representations. Meryansumayeka et al.[14] illustrated design research on digital task design which aims at developing higher order thinking skills (HOTS) for junior high school students. The

digital tasks were developed based on the PISA problems and were validated and evaluated for the field test. Zhang et al.[15] posited that the variation in task design has a profound theoretical foundation and developed an analytic model of "teaching mathematics through variation" which distinguishes potential variation and practical variation. The model illustrated a teacher's documentation work from potential to practical variation in in China and France. Jäder[16] addressed how students' opportunities to engage in mathematical problem solving is limited by the prevalence of routine tasks in textbooks. An analytic framework was developed to better understand some of the important components of mathematical problem solving and possibly also be of support in the design of mathematical problems. Doorman et al.[17] discussed task design with scenarios in promoting inquiry-based mathematics teaching. The combination of the RME and the TDS afforded the development of open and context-rich tasks and to support teachers in balancing phases of student-led inquiry with phases for creating a whole class shared understanding of mathematical structures. Dawn et al.[18] proposed four interacting elements for consideration when developing a mathematical activity to support the construction of mathematical concepts. Analysis of students' work from the activity revealed different trajectories of how teachers can plan for differentiated instruction to promote students' robust construction of concepts.

2.5. *Session 4*

In the session 4, we had seven short oral presentations. Rodriguez et al.[19] prezented a sequence of tasks to help university students to go from an elementary conception of vector in physics to an element of a vector space. The task sequence involved a contextual problem in a digital environment with increasing abstraction. It began with movement of a robotic arm then as an arrow with magnitude and direction, and finally as an ordered pair of real numbers in a geometric environment. Nagari-Haddif[20] demonstrated design pattern of tasks using the Seeing the Entire Picture (STEP) online assessment platform in which students were to construct and submit examples for refuting or supporting a statement in an MLR environment with the activity "asymptotes and parametric functions". Nieto and Ruiz-Lopez[21] reported design reserch on the creation of a didactic sequence of problems and a technological tool to guide the resolution for teaching and learning isometries for prospective primary teachers. Mohamed and Ghazali[22] devised fraction tasks and test items which were validated by experts. Through these instruments and the Newman procedure types of errors concerning fractions were identified. Falculan and Aberin[23] investigated the effects of using Low Floor High Ceiling (LFHC) mathematical tasks on students' mathematical proficiency in seventh-grade geometry by closely examining their conceptual understanding and procedural fluency. Ohtani[24] presented an activity-theoretical approach to collaborative design of task and learning environment in which researchers of different expertise, and secondary teacher play different roles.

3. Recent Trends and Future Directions

We laid out seven themes in section 1.1. In view of the seven related themes, we point out recent trends and pose future directions for further research that arose from the TSG.

1) frameworks and principles for task design

We had many presentations that evidenced productive coordination and/or joint of different frameworks and principles. Among others, emergence of the embodiment perspective demonstrates promise for furthering task design. Chinese variation theory also drew much attention in the TSG. We need to investigate further how different design principles reflect or generate different perceptions of mathematical concepts.

2) methodological advances for studying task design in mathematics education

Many presentations adopted Design Research methodology, including the iterative processes of thought experiments and teaching experiments. The model and modelling perspective were also prevalent in the TSG presentations. In view of the wide interest in STEM movement, it is crucial to incorporate methodological advancements from other disciplines.

3) relationships between task design, anticipated pedagogies, and student learning

In this theme, recent research emphasizes inquiry-based learning, study and research pass, and cooperative learning. In this regard, it is necessary to reflect how different combinations of tasks and pedagogy influence learners' perceptions and mathematical activity.

4) the role of tools in task design

Some research developed digital tasks and video tasks. It is also relevant topic in task design and analysis to investigate how visual features of task presentation affect mathematical activity.

5) task sequences for promoting conceptual understanding and/or higher order thinking skills

In our knowledge-based society students need so called 21st century skills. A way to create opportunities for addressing these skills in classroom is through inquiry-based learning (IBL). In the TSG, we had promising presentations that suggested the design of appropriate tasks could be an important prerequisite for successful implementation of IBL.

6) task design in innovative learning environments

The development of technologically rich environment and assessment system enables us to understand the complex relationship between task design and individual learner differences.

7) textbook task analysis

This theme was not listed in the discussion document of the TSG. However, we had several presentations that compare textbook task in different countries, specifically comparison of Chinese and European countries.

In addition to the aforementioned seven themes, the following themes were prevalent in the TSG.

8) teachers' professional learning on task design and analysis

The professional learning of prospective and practicing teachers about task design, sequencing, and adaptation. This emerging theme, which goes beyond superficial aspects, would require socio-cultural perspectives in task design.

9) notworking among stakeholders

The communities involved in task design are naturally overlapping and diverse and they may act in several of these roles. Stakeholders can include designers, professional mathematicians, teacher educators, teachers, researchers, learners, and policymakers and so on. The effectiveness of forms of collaboration and communication between task designers, classroom teachers, educators, and policymakers.

10) political and ethical dimension of task design

This theme was less obvious in the TSG. The role of task design in promoting equity and other values are less attentive but relevant theme which should be considered in future research.

Looking to the future, TSG participants who are examining interdisciplinary efforts, instructional tools, online resources, and research methodologies could expand their vision by sharing empirically grounded contributions that underlie design principles, theoretical approaches, and carefully analyzed cases and examples of tasks designed for promoting mathematical development. Another area to explore are the multi-dimensional aspects of task design: tasks and sequences of tasks can shape possibilities for interactions between teachers and students. Teachers' pedagogies can include the selection, modification, design, sequencing, installation, observation and evaluation of tasks, through which they may learn more about their students' thinking and experiences. In turn, students' interactions with tasks can afford opportunities to learn mathematical concepts, ideas, strategies, and also to use and develop higher order thinking skills and critical literacy.

References

J. B. Ärlebäck and L. Albarracín (2019). The use and potential of Fermi problems in the STEM disciplines to support the development of twenty-first century competencies. *ZDM: The International Journal on Mathematics Education*, *51*(7), 979–990.

M. Artigue (2014). Didactic Engineering in mathematics education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 159–162). Cham, Suiza: Springer.

J. P. Becker and S. Shimada (1997). *The open-ended approach: A new proposal for teaching mathematics*. Reston, Virginia: National Council of Teachers of Mathematics.

M. Bosch (2019). Study and research paths: A model for inquiry. In B. Sirakov, P. N. de Souza, & M. Viana (Eds.), *Proceedings of the International Congress of*

Mathematicians: Rio de Janeiro 2018 (Vol. 3, pp. 4001–4022). Singapore: World Scientific Publishing.

G. Brousseau (1997). *Theory of didactical situations in mathematics. Didactique des mathématiques 1970–1990.* Kluwer Academic Publishers, Dordrecht.

Y. Chevallard (2015). Teaching mathematics in tomorrow's society: a case for an oncoming counter paradigm. In S.J. Cho (Ed.), *The Proceedings of the 12th International Congress on Mathematical Education* (pp. 173–187). Dordrecht, The Netherlands: Springer.

C. Duijzer, M. Van den Heuvel-Panhuizen, M. Veldhuis, M. Doorman, and P. Leseman, P. (2019). Embodied learning environments for graphing motion: A systematic literature review. *Ed. Psych. Revw.*, *31*, 597–629.

Y. Engeström (2014). *Learning by expanding: An activity-theoretical approach to developmental research*. New York: Cambridge University Press.

S. Evans and M. Ayalon (2016). Can designed student responses support teachers to interact with students in a productive way? *Educational Designer*, *3*(9), 13.

H. Freudenthal (1991). *Revisiting mathematics education*. Dordrecht: Kluwer.

G. Gueudet and L. Trouche (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics*, 71(3), 199–218.

K. Hoogland, B. Pepin, J. de Koning, A. Bakker and K. Gravemeijer (2018). Word problems versus image-rich problems: an analysis of effects of task characteristics on students' performance on contextual mathematics problems. *Research in Mathematics Education*, *20*(1), 37–52.

H. L. Johnson, A. Coles, and D. Clarke (2017). Mathematical tasks and the student: navigating "tensions of intentions" between designers, teachers, and students. *ZDM: The International Journal on Mathematics Education*, *49*(6), 813–822.

N. Kozcu (2017). Effect of 5e learning model on academic achievement, attitude and science process skills: Meta-analysis study. *Journal of Education and Training Studies, 5*(11), 157–170.

J. Lithner (2017). Principles for designing mathematical tasks that enhance imitative and creative reasoning. *ZDM: The International Journal on Mathematics Education, 49(6)*, 937–949.

A. Oktac, M. Trigueros, and A. Romo-Vázquez (2019). APOS theory: connecting research and teaching. *For the Learning of Mathematics*, *39*(1), 33–37.

X. H. Sun (2016). *Spiral variation: A hidden theory to interpret the logic to design Chinese mathematics curriculum and instruction in mainland China*. Singapore: World Scientific.

M. Swan, D. Pead, L. M. Doorman, and A. H. Mooldijk (2013). Designing and using professional development resources for inquiry-based learning. *ZDM — International Journal on Mathematics Education 45* (7), (pp. 945–957) (13 p.).

A. Wijaya, M. van den Heuvel-Panhuizen, and M. Doorman (2015). Opportunity-to-learn context-based tasks provided by mathematics textbooks. *Educational studies in Mathematics*, *89*(1), 41–65.

A. Watson and M. Ohtani, M. (Eds.). (2015). *Task design in mathematics education: An ICMI study 22*. New York: Springer.

M. Yerushalmy, G. Nagari-Haddif, and S. Olsher (2017). Design of tasks for online assessment that supports understanding of students' conceptions. *ZDM Mathematics Education, 49, 701–716*.